A Review of Computer Simulations for Aircraft-Surface Dynamics

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Introduction

URING the design of an aircraft heavy emphasis is placed on flight-induced motions and loads. However, groundbased operations produce an environment that can generate significant aircraft dynamics uncomfortable to passengers or damaging to the cargo. In addition, high vertical accelerations in the cockpit represent a potential disorientation problem for the pilot, which may cause landing or takeoff accidents. Perhaps more important, the aircraft structure can be subjected to large local deformations leading to stress failure, or the gears could experience loads beyond their design limits. For normal commercial aircraft and airports, ground loads (except for landing impact) should be of secondary concern. But aircrafts such as crop dusters and small private planes, which often operate from unimproved fields, experience a harsh environment during ground operations. Of greatest importance is the growing need of military aircraft to be operational from austere airfields.

Most current U.S. Air Force (USAF) aircraft operate on rigid, smooth, paved Main Operating Base (MOB) surfaces. In all of the recent wars that the United States has been involved in, it has enjoyed air superiority, and its airbases were generally well protected and operational under normal procedures. Future conflicts may, however, be fought from MOBs vulnerable to enemy attack, and MOB surface damage or MOB denial is anticipated. Therefore, the USAF is placing greater emphasis on aircraft-surface operations, particularly on bomb damaged repaired (BDR) surfaces, soil, and other emergency surfaces.

One effort to meet this challenge, which is presently under way in the United States, is to define the rough surface capabilities of mainline fighters and cargo planes. The USAF is establishing these capabilities through a program called HAVE BOUNCE¹ under which:

- 1) Simulations are prepared for each aircraft on BDR runways.
- 2) Aircraft component weaknesses are identified through simulation.
 - 3) Simulations are validated with test data.
 - 4) Operational limitations are developed.

In the past 5-10 years a substantial number of computer simulations have been developed to predict aircraft-surface in-

teraction. Many of these programs have been written by USAF personnel or have been contracted to various organizations by the USAF. Others have been developed by aircraft companies to meet their own needs, or by individuals at universities, or in foreign countries (most notably in NATO countries).

The objectives of this study were to review the literature concerning aircraft-surface dynamic simulation techniques: 1) to establish a historical view of the improvement in the state of the art, 2) to recognize the individuals and organizations that have played a prominent roll in advancing the state of the art, 3) to develop a knowledge base of physical phenomena that have been simulated, 4) to identify mathematical techniques that have been used, 5) to classify the simulations according to their general purpose, complexity, and accuracy, and 6) to suggest areas in which simulation techniques could be improved, and test could be run to validate the simulations.

This report contains a brief summary of the computer programs written to predict the dynamic displacements and forces resulting from nonflight aircraft operations. The capabilities of each program along with their limitations and numerical techniques are cited.

Specific Features

Degrees of Freedom (DOF)

Central to any dynamic analysis is the extent to which the motions of the system are included in the model, i.e., which DOF are necessary to accurately predict the phenomena being investigated. Typically, the airframe has been modeled using as little as the bounce (vertical) DOF to six rigid-body DOF, and several symmetric or asymmetric flexible modes. The landing gears are independent units on the aircraft and each one can be modeledwith one to six rigid-body DOF. In some simulations the wheels are given DOF separate from the landing geats to investigate spin-up, braking, and other wheel/surface interaction. Finally, there have been a couple of simulations that consider the cargo or propulsive devices as separate DOF.

Strut Suspension

The connection between the airframe and landing gears is provided by a suspension device. In the simple models the

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device is a linear spring and damper in parallel, Fig. 1a. More realistic struts involve nonlinear stiffness, Fig. 1b, and velocity squared dampers, Fig. 1c. In addition, the damping coefficient may be a nonlinear function of the stroke of the strut. Also, friction/stiction may be modeled in the strut. A few simulations have included active suspensions.

Tires

The most common tire model is the point-contact follower, Fig. 2a, with either linear or nonlinear stiffness and damping in parallel. Recent models have been developed to better approximate the enveloping effect of the tire traversing short wavelength obstacles. The radial spring tire, Fig. 2b, with either linear or nonlinear stiffness elements, has been used in several models.

Surfaces

Most simulations have been written for motion over rigid surfaces. In some models any surface elevation is possible, while in others the model is specifically designed to simulate the aircraft dynamics over a discrete obstacle, e.g., a double $(1-\cos)$ bump representing BDR runways, Fig. 3. Yielding surfaces (soil) are typically represented by some combination of linear or nonlinear springs and dampers, Fig. 4. Those soil models with a damper in series with other soil elements are used to predict soil rutting.

Simulations

A literature search and review of aircraft-surface dynamics was conducted by Cox et al.2 in 1979. They gave a general discussion of input characteristics for simulations, the types of modeling techniques used, the outputs of the models, solution techniques, and the relation of automobile and rail dynamics to aircraft dynamics. Only five aircraft-surface simulation codes were mentioned. Drevet³ developed a simulation on an analog computer to investigate aircraft takeoff. Wignot et al.4 included airframe flexible degrees of freedom in a digital program to study dynamic loads during taxiing. Boozer and Butterworth⁵ used a flexible aircraft with nonlinear landing gear and tire stiffnesses to study the dynamic response of a C-141A taxiing on a rough runway. A follow-on computer code by Gerardi and Lohwasser⁶ increased the complexity of the gear model, but attempted to keep the simulation as simple and versatile as possible so that any aircraft could be simulated at a reasonable cost. The preceding two programs predicted the aircraft dynamics and strut forces. Kilner's⁷ simulation predicted aircraft component loads in an attempt to establish acceptable BDR techniques. It is important to note that all of the above aircraft-surface programs were for pitch plane dynamics only, and that no attempt was made to model the soil as a viscoelastic element. Cox et al.2 also made the point that the pilot was not included in the simulations (open-loop).

Many of the aircraft-surface computer codes have been written in the past 5 to 10 years to satisfy the HAVE BOUNCE program, but many computer codes, in addition to those reviewed by Cox et al.,² existed prior to this major program.

In 1962, Silsby⁸ used a simple linear model to determine the power spectrum acceleration response of a rigid-frame supersonic transport due to the power spectrum of runway unevenness. The aircraft had two degrees of freedom—pitch and bounce—and the undercarriages each had a bounce degree of freedom. All damping and stiffness characteristics of each strut were linear and identical. Silsby's results were strongly and inconsistently influenced by speed; but he did conclude that the acceleration response at the cockpit of a supersonic transport would be worse than that of a subsonic transport.

Tung et al.⁹ developed a ten-degree-of-freedom digital simulation to investigate supersonic aircraft vibrations when traversing runway intersections and other unevennesses. The model included rigid-body bounce and pitch of the airframe and bounce of two landing gears, six flexural modes of a free-free airframe, and nonlinear damping and stiffness

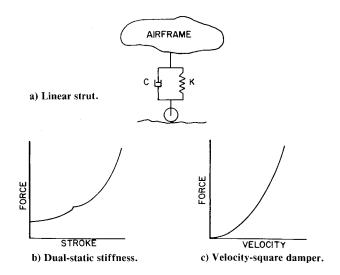
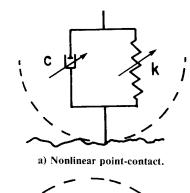
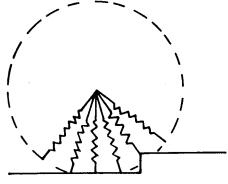


Fig. 1 Strut characteristics.





b) Radial spring tire.Fig. 2 Tire models.

characteristics, with friction, in the struts. The equations were numerically integrated to determine the vertical acceleration of the pilot. The results indicated that supersonic aircraft, with a long slender fuselage, produce significantly higher cockpit accelerations than subsonic aircraft, and that the flexural modes greatly increased the cockpit acceleration.

Ortasse¹⁰ suggested a different approach in constructing a deterministic runway profile with the same spectral content as several representative measured profiles. Although he discussed the importance of airframe roll and asymmetric modes, his model was pitch plane only with rigid-body pitch and bounce and four symmetric flexible modes. Undercarriage nonlinearities were discussed, as well as aerodynamics and structural damping. His theoretical results showed good agreement with measured data at low taxi speeds, but inaccuracy increased with speed.

Bolton et al.¹¹ developed digital and hybrid computer simulations for both deterministic and random runway input to predict undercarriage and airframe loads on the Boeing 747. Their deterministic model was three-dimensional and in-

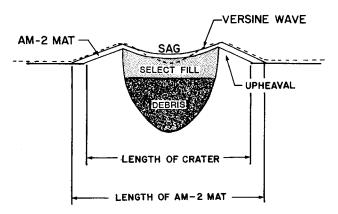


Fig. 3 Bomb damage repaired runway obstacle.

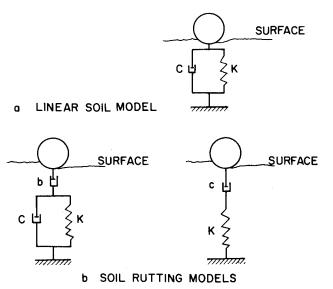


Fig. 4 Soil models.

cluded the nonlinearities of the oleopneumatic† struts. Runway inputs were 1-cosine waves. The input for the random analysis was spectral densities for commercial runway profiles. All loads were below the design load levels.

In 1968 Richmond et al. 12 developed an analog model for taxiing, and digital models for landing and takeoff, specifically to analyze a Boeing 367-80 on substandard runways. The model included three airframe rigid-body modes (bounce, pitch, and roll), and three symmetric and two asymmetric airframe flexible modes. The landing gear degrees of freedom included bounce and longitudinal springback for each of the three gears, and truck pitch for the two main gears—a total of 16 degrees of freedom. The gears were oleopneumatic; the tires were nonlinear springs with a point-contact follower; the soil was a nonlinear, rate-sensitive spring; and the runway obstacle was modeled by 1-cosine dips or bumps.

Sharp¹³ used the same test results as Richmond et al.¹² on the Boeing 367-80 to verify a one-degree-of-freedom simulation for C-141 and C-5 takeoffs on clay or sand airfields. Although Sharp's wheel/soil interaction was very detailed, none of the strut characteristics were included. Much of the wheel/soil interaction model was based on empirical data. Sharp used a fourth-order Runge-Kutta with a variable time step for numerical integration.

Furnish and Anders¹⁴ developed a three-dimensional model of a flexible aircraft: bounce, longitudinal, pitch, roll, yaw, and up to 10 coupled flexible modes. The emphasis in this model was the gear, which included: polytropic air compression, velocity squared damping, variable orifice shape, snubber orifice damping, and strut friction with breakout and lockup. This model was verified by mounting an A-37B aircraft on hydraulic actuators and simulating a 1-cosine bump.

A statistical approach was used by Kirk and Perry¹⁵ and Kirk¹⁶ in which they considered only the bounce degrees of freedom of the airframe, wheel, and the first symmetric wing bending mode. The strut stiffness and damping were linearized, and the input was defined as the vertical spectral density of the runway. Transfer function techniques were used to predict rms displacements and forces. An extension to the statistical model was made by Kirk¹⁷ when he added the pitch rigid-body mode, but he removed the flexible mode used in his first model. Furthermore, to simplify the analysis, he uncoupled the bounce and pitch motions by assuming that the mass/stiffness relationships of the nose and main struts were identical. Comparison to experimental data from the Boeing 707 differed by 18% at the center of gravity and 10% at the cockpit.

Hsueh, 18,19 in his doctoral thesis, developed time- and frequency-domain simulations of the pitch plane dynamics of a flexible aircraft during ground operations. Rigid-body degrees of freedom included fuselage bounce and pitch, and nose and main gear bounce. Symmetric flexible modes were also included. The tires were represented by linear springs, and the landing gear mechanism by a parallel combination of a nonlinear spring, a nonlinear dashpot, and a Coulomb friction device. Aerodynamic lift was included. Numerical integration was performed by a finite difference method. Comparisons of the variance in vertical accelerations at the cockpit and center of gravity, and main and nose landing gear tire forces were made for a Boeing 707 airplane with rigid-body modes only and with the flexural modes added. The flexural modes decreased the tire forces and the center-of-gravity acceleration, but generally increased the cockpit acceleration.

A master's thesis by Corsetti²⁰ simulated a C-130 aircraft with active hydraulic controllers in the struts. The analysis was stochastic; and the performance criterion was to reduce wing fatigue damage. Three separate models were developed with the most complex model including airframe bounce, pitch, and roll along with wing flexibility represented by an additional spring/mass/damper arrangement. Results showed that wing fatigue damage would be reduced by optimizing the active controllers in the struts.

Lynch et al.²¹ developed a general six-degree-of-freedom simulation for a rigid airframe with up to five landing gears. The program could simulate control and performance during glide slope, flare, landing, or takeoff, all under conditions of winds and braking, or strut or engine failure. It also modeled ground effects, engine reversal, drag chute, carrier takeoff, inclined runways, runway perturbations, landing gear loads, and control systems. A fourth-order Runge-Kutta integration technique was used with variable step size leading to a long solution time.

Mitchell²² developed a two-dimensional model to study the cockpit acceleration of the Concorde during taxiing. His model included the bounce and pitch rigid-body modes plus the first eight symmetric modes of the airframe. The struts were modeled with nonlinear stiffness, damping, and friction and stiction. Aerodynamics were included. Mitchell concluded that taxi performance could be improved by significant reductions in the strut stiffnesses.

Whitehead²³ presented a thesis in which he developed a twodimensional hybrid simulation of an aircraft taxiing over deterministic runway profiles. The model assumed two rigidbody (bounce and pitch) and five symmetric flexible modes for the airframe, and one bounce mode each for both the nose and main undercarriages. The gears contained nonlinear stiffness,

[†]Oleopneumatic implies a nonlinear force-stroke relation involving a polytropic process of compressing a gas, and a nonlinear forcestroke velocity relation involving the forcing of a hydraulic fluid through an orifice.

damping, friction and stiction, and the tires were assumed to have linear stiffness and damping with point contact. Aerodynamic lift and moment were included. Whitehead proposed an optimization technique based on ride comfort and structural fatigue to determine the undercarriage parameters. The simulation time histories were compared to Mitchell's²² results for the Concorde for validation.

A complex three-dimensional model of aircraft dynamics on a runway was developed by Reynolds.²⁴ The model included symmetric and asymmetric flexible airframe modes, all of the nonlinearities for any number of undercarriages, and ground effect aerodynamics. A Runge-Kutta-Merson numerical integration technique was used to obtained solutions. The main result of the study was that the model was impractical due to long computer times.

In 1975, Crenshaw²⁵ developed a series of five computer programs to investigate soil/wheel interaction and aircraft response during landing, taxiing, takeoff, and turning. The first program predicted the number of passes by an aircraft over a soil before a certain rut depth was produced. The rutdepth formula was based on empirical results developed by Waterways Experiment Station,²⁶ and was good for low-speed operation only. The second program computed vertical wheel load during landing impact on a yielding surface, and the rut depth due to the vertical and spin-up loads. In this dynamic simulation the ground model was a primary soil spring in series with a secondary soil spring and a viscous damper in parallel. Wheel slip and soil hardening effects were included. The airframe had two degrees of freedom (bounce and pitch) and was acted upon by aerodynamic lift and strut forces. The struts were modeled as single-degree-of-freedom oleopneumatic elements with no bearing friction. The tires' loaddeflection characteristics were nonlinear. Because of the high frequency of the unsprung mass, a fourth-order Runge-Kutta integration was used. The third computer code included landing impact and runout with cyclic braking.

The fourth program developed by Crenshaw²⁵ simulated taxi and takeoff in the pitch plane for an aircraft with up to five landing gears. At the beginning of taxiing, static balance was achieved by iteration. Aerodynamic lift and drag, but not moment, were included, as well as thrust as a function of velocity. The same soil and landing gear models that were used for landing were used for the taxi and takeoff simulations, but the numerical integration technique used was a Taylor series rather than Runge-Kutta. The fifth program simulated turning using the landing gear and soil models developed for the landing and takeoff programs. After a short taxi time, the nose gear was turned such that the wheel had both free rolling and skidding components. To accommodate the turning effect, two additional airframe degrees of freedom were assumed for a total of four: bounce, pitch, lateral, and yaw. Taylor series, integration was also used in the turning simulation.

Durham and Murphy²⁷ used the U.S. Army's vehicle dynamics model to predict the pitch plane response of a C-12A on substandard runways. Four rigid-body degrees of freedom were used: body bounce and pitch, and nose and main gear bounce. The nonlinear stiffness and damping characteristics of the struts were modeled; and the tires were assumed to be clusters of radial springs.

An aircraft simulation by Gerardi and Lohwasser⁶ was a pitch plane analysis of aircraft response during a takeoff roll over a runway, approximated by a third-order polynomial at elevations specified every 2 ft. The degrees of freedom included: longitudinal, pitch and bounce, and 15 flexible modes for the airframe; and the nose and main gears' bounce. The struts were oleopneumatic (friction neglected), and the tires were linear point-contact springs. The equations of motion were integrated using a three-term Taylor series. In 1977 a roll degree of freedom was added by Gerardi, ²⁸ along with 15 asymmetric airframe flexible modes, and asymmetric runway profiles.

A digital program was developed by Kilner²⁹ specifically to simulate the dynamics of the F-4C and F-111 aircraft taxiing over BDR runways. Somm et al.³⁰ used a modification of the program to analyze the T-43A, KC-135, and YC-14. The model simulated the bounce and pitch modes of the airframe, and the bounce of the nose and main wheels along with symmetric flexible modes (8 for the F-4C and 15 for the F-111). Aerodynamic lift and moment were applied, and used to calculate loads on the fuselage and wings. Strut forces included pneumatic springs, hydraulic damping, stops, and friction. Wheel drag loads were also calculated. A dynamic programming language, MIMIC, was used with FORTRAN subroutines added for initial conditions and I/O variables. A Runge-Kutta technique will automatic time-step adjustment integrated the equations of motion. The simulation predictions were used to establish aircraft component failure criteria.

The general capabilities of a digital simulation used at McDonnell Douglas are summarized by Burkhart and Wilson.³¹ This time-domain simulation assumed four rigid and up to ten symmetric, flexible-body degrees of freedom for the airframe. Strut oleopneumatic and friction forces were modeled for up to five flexible landing gears—each with six degrees of freedom. Thrust, braking, and control surfaces could be varied based on taxiing or landing simulations. Aerodynamics and ground friction were also modeled. BDR runway profiles were simulated by ramps and constant curvature arcs. A predictor-corrector technique with a constant time step was used to numerically integrate the equations of motion. Numerous computer runs indicated potential structural problems in operating the F-4C and F-4E on BDR runways.

The EASY computer program is basically an analytical tool originally developed by Boeing to model control systems. As such, the program has many capabilities, e.g., root locus, eigenproblem, and stability. The code consists of modules that must be assembled in the model generation part of the program. The model is then analyzed based on the desires of the user. In 1979, EASY was expanded to EASY-ACLS (Air Cushion Landing System). Aircraft equations of motion, various air cushion devices, and arresting gear capability were added. The model generator could select up to six rigid-body degrees of freedom, thrust variations, and wind gusts. The program could simulate landing, takeoff, taxiing, and flight. Integration techniques available were Runge-Kutta (fixed or variable step), Adams/Bashforth/Moulton predictor-corrector, Euler, Heun, and Gear.

A second revision to EASY was made in 1980 when Warren and Kilner added an Advanced Brake Control System.³³ This new version of the program was developed to simulate aircraft-surface response during adverse weather conditions. The program included pilot input in the form of control surfaces, throttle, and brakes. Each strut had four degrees of freedom: bounce, longitudinal, lateral, and steering. The tire characteristics were nonlinear properties of displacement. The ground profile (different for each wheel) could be sinusoidal or random.

A different approach to rigid-body aircraft simulation was taken by Gajewski³⁴ when he developed the total simulation model (bounce, pitch, and roll) based on modal equations. Knowing natural frequencies, damping ratios, and mode shapes, he integrated the equations of motion using the Newmark or Wilson θ method. Symmetric or asymmetric surface roughness on a rigid runway was used to excite the aircraft. The strut and tire stiffnesses were combined in series, and the wheel masses ignored.

Dawson and Larkins³⁵ developed a pitch plane dynamic simulation for the F-4E traversing runway irregularities including AM-2 mats. The airframe had 3 rigid-body modes—longitudinal, bounce, and pitch—and 15 flexible modes for the wing, fuselage, and pylons. Each nose and main gear had a bounce mode, and the struts were oleopneumatic

with rebound snubbing and friction. The tire was modeled as either a nonlinear spring with point contact or a pneumatic membrane that enveloped surface irregularities. Thrust and aerodynamic lift, drag, and pitching moment acted at the aircraft's center of gravity, and were used to calculate loads on the wings and fuselage. This model had an unusual feature that would simulate the effect of tail scrape. An iteration process was used to balance the aircraft for initial steady-state conditions, and a Runge-Kutta numerical integration with variable time steps solved the equations of motion.

A Master's thesis by eight graduate students at the Air Force Institute of Technology resulted in a three-dimensional simulation of an aircraft operating over a BDR runway. ³⁶ The primary purpose of the simulation was to investigate one active and three passive alternatives to the current F-16 shock strut. The airframe was modeled with 5 rigid degrees of freedom (bounce, pitch, longitudinal, lateral, and roll), and up to 20 symmetric flexible modes. Each strut had a bounce degree of freedom and was modeled as a nonlinear spring and damper in parallel. The tire was modeled as a nonlinear spring having point contact with the surface. The surface was assumed to be rigid with roughness elevation modeled by algebraic or trigonometric expressions. The Adam's predictor-corrector was used to integrate the equations of motion.

A seven-degree-of-freedom model by Ottens and Nederveen³⁷ was part of the Netherlands' contribution to NATO's investigation of aircraft response induced by runway bumps or repairs. The model included the bounce, longitudinal, and pitch motion of the airframe, and the bounce and longitudinal motion of the nose and main gears. Landing struts were represented by oleopneumatic sliding members and a linear torsional spring. Tire stiffness was a function of deflection, and point contact was assumed at the tire/surface interface.

Work done in Czechoslovakia by Kropac et al. 38 produced a frequency-domain computer program that predicted the vertical vibrations of an aircraft excited by runway unevenness and propulsion forces. The results were interpreted as comfort criteria for the crew and passengers and for structural safety. The model included five bounce degrees of freedom: sprung mass, unsprung mass, flexible-body sprung mass, passenger or cargo mass, and the mass of a propulsion device. Linear elastic and damping elements were used to connect the different masses, and the tire was represented by a linear spring.

Much of England's contribution to NATO's investigation of military aircraft response on damaged and repaired runways was summarized by Payne et al.^{39,40} While their stated goal was to use relatively simple general-purpose computer programs, many important details were considered. The models contained both rigid and flexible modes for the fuselage; polytropic gas spring, velocity-squared damping, and friction/stiction in the gears; tire models; undercarriage details; and even parachute deployment. A Kutta-Merson variable-step integration resulted in a significant savings in computer time compared to a fixed step. Several aircraft were modeled (e.g., Concorde, C-130, VC-10, Jaguar, etc.), and excellent correlation with measured data for low speeds was made.

An initial examination of the suitability of hand-held programmable calculators to predict aircraft response was made by Taylor et al. ⁴¹ Both a TI-59 and a HP-41C were used. In particular the TI-59 was used to integrate the bounce degree of freedom of an aircraft traversing a runway repair mat. Stiffness and damping forces were linear, and a Runge-Kutta numerical integration was used. Lack of memory limited the analysis to one degree of freedom, but the gear load response correlated well with Gajewski's³⁴ model which also ignored wheel mass.

Cook⁴² developed a computer simulation that was simple in terms of only three degrees of freedom, but complex in the tire/soil interaction. The program simulated a single rigid landing gear supporting an effective aircraft mass. The degrees of freedom were bounce, longitudinal, and wheel spin.

Surface roughness was approximated by a fourth-order polynomial, while soil flexibility was modeled as a nonlinear spring and damper in series. The tire was modeled as radial springs whose force dependency on displacement was quadratic. An iteration procedure was used to determine the forces in the tire and soil at each time step. Integration was performed using a Taylor series. The program could predict tire sinkage for a static aircraft, and then axle startup loads due to thrust buildup and subsequent motion.

Due to the need for a short takeoff capability, the U.S. Navy has investigated ramp-assisted takeoff. ⁴³ The model included bounce, pitch, and longitudinal for the airframe, and bounce for the nose and main gears. The struts were oleopneumatic, and the tires were nonlinear point-contact followers with rolling friction. Detailed aerodynamics, controlled by pilot input to control surfaces, and thrust as a function of velocity, throttle setting, and air temperature, were modeled. The ramp was either circular or user-supplied. The fourth-order Runge-Kutta technique was used to integrate the equations of motion. A wasteful feature of this simulation was that it integrated the lateral, yaw, and roll equations of motion even though there were no motions in those directions.

A third major modification by Skinner et al. 44 was made to Gerardi's simulations. 6.28 The new program, TAXIG, was a modularization of the old ones for the purpose of making it more flexible to model any aircraft. The new program used a Newton-Raphson iteration to balance the aircraft and produce initial conditions. In addition to the Taylor series integration technique, the program could also use the Adams-Moulton predictor-corrector with a Runge-Kutta starter. Skinner's modifications included new capabilities such as simulating landing impact, a variety of landing gear geometries, and braking and aerodynamic devices such as tailhook or drag chute.

A one-dimensional simulation of an aircraft under the actions of weight, aerodynamic lift and drag, soil drag, and thrust was developed by Phillips et al. ⁴⁵ Although this model had only one degree of freedom (longitudinal), it did contain Cook's tire/soil model, ⁴² which was a tire with quadratic radial springs, and a damper and spring in series for the soil. The program predicted sinkage and drag during takeoff or landing on soil. It used a fourth-order Runge-Kutta numerical integration with a variable time step.

Levy46 summarized work that had been done at Fairchild Republic Company to analyze the pitch plane dynamics of the A-10 aircraft traversing both deterministic and stochastic runways or soil. The main emphasis was to study six conceptual landing gears and their effect on gear/wheel loads, ground loads, tire deflection, and rut depth. The soil was modeled with a linear spring in series with a combined parallel linear spring and velocity damper. The tire was represented by "n" radial springs with quartic and cubic force-displacement functions, based on a model at the University of Dayton. 42 Articulated and active landing gears and antiskid brakes were modeled. The degrees of freedom included rigid-body airframe longitudinal, bounce, and pitch; wing bending and torsion; fuselage bending; stores lateral and pitch; and bounce, longitudinal, and pitching of struts. Time-domain studies were done with a second-order Taylor series numerical integration. One interesting study was made to determine when the multielement tire model would yield significantly different results than a single-point tire model. Multielement tires gave higher loads, especially for small-scale irregularities at low to moderate taxi speeds.

Crenshaw and Hollenbeck⁴⁷ developed a three-dimensional model to investigate a F-4C fighter taxiing over a soil surface of varying strength. This simulation, in particular, included turning in soil. The airframe had six, while each wheel had one, rigid-body degrees of freedom. In addition, each strut had a flexible bending mode that affected the gear load, but did not affect the airframe dynamics. The strut suspensions

were oleopneumatic. The tire was modeled by a point-contact follower or a distributed contact (radial springs). ⁴² The soil was modeled in the vertical direction as a spring in series with a spring and damper in parallel. In the longitudinal direction a free rolling wheel with rolling drag, and in the side direction an element that included bulldozing drag and wheel skid was assumed. Due to both soil and tire flexibility, an iteration was necessary to determine soil deflection; the Method of False Position was used. The equations of motion were originally integrated using a fourth order Runge-Kutta, but the program has undergone modification to Adam's predictor-corrector. ⁴⁸

Cook used his previously developed, ⁴² detailed tire/soil interaction model, and expanded it to a complete three-dimensional aircraft model. ⁴⁹ The airframe had six rigid-body degrees of freedom. Each of three wheels had a spin degree of freedom that included braking and slip effects; and each strut was modeled as oleopneumatic with sliding friction. The tires were modeled with quadratic radial springs, the soil with a spring and damper in series, and the surface profile with a fourth-order polynomial. Pitch plane aerodynamics and thrust through the center of gravity were included. The program could be used in three modes of operation: takeoff; landing; and stop, sink, and startup. Numerical integration was performed by a Taylor series.

A more complete simulation by Taylor et al. 41.50 was developed to run on an Apple II computer. The model included the rigid-body bounce, pitch, and roll degrees of freedom of the fuselage, the bounce degree of freedom of up to five gears, and up to five symmetric airframe flexible modes. Gear characteristics were represented by oleopneumatic elements, and the tires were linear point-contact springs. Five numerical integration techniques were available: fourth-order Runge-Kutta, Houbolt's, central differences, Taylor series, and Newmark. The model could be used to simulate either three or five post aircraft traversing single or double runway repair mats. This model, originally developed for an Apple II computer, was made operational on a VAX and used to investigate the response of the F-16 to ramp inputs.

As part of the HAVE BOUNCE program, Crenshaw and Owen⁵¹ modified and improved existing simulations to predict C-130 aircraft dynamic response and loads when traversing AM-2 mats. Although the program contained the airframe roll degree of freedom, only bounce and pitch motions were simulated over symmetric obstacles. The first eight flexible modes due to wing bending and torsion were also used. The strut models contained the oleopneumatic nonlinearities plus bearing friction, while the tires were point-contact, nonlinear springs with linear damping. Each wheel axle had a bounce degree of freedom. Control inputs for elevator deflection, thrust, and braking were provided. Aerodynamic lift and moment as functions of ground speed and aircraft attitude were a major development in the simulation. The computer simulations were compared to test data, and the program was modified, resulting in reasonable accuracy in peak load predictions. The equations of motion were numerically integrated using a fourth-order Runge-Kutta technique.

Also as part of the HAVE BOUNCE program, Justice⁵² developed an aircraft-surface simulation for the C-141B cargo plane. The model included 4 rigid-body degrees of freedom for the airframe (bounce, longitudinal, pitch, and roll) and 15 airframe flexible modes. The three struts were oleopneumatic with sliding friction. Each strut also had a springback degree of freedom. The tires were represented by nonlinear springs, and interacted with the ground as point contacts. Runway roughness simulated AM-2 mat configurations by using ramps and flats or 1-cosine bumps. Pitch plane aerodynamics and thrust as a function of velocity were included. The simulation was capable of landing impact, landing and takeoff runout and taxiing. The equations of motion were numerically integrated using the Runge-Kutta-Gil technique. This basic computer simulation was also used by Kent et al.53 to simulate the C-5A for the HAVE BOUNCE program.

A project recently completed by Northrop⁵⁴ was to develop a computer simulation to combine airframe structural modeling with the latest advances in tire, soil, and landing gear modeling. This program-NORTAX-was an extension of TAXIG44 and is generally applicable to any military aircraft regardless of landing gear type. Because the program was written in modules, the user can select different components to solve a particular problem, or can write his own module. The airframe had 6 rigid-body degrees of freedom and up to 15 flexible modes. Each gear had four degrees of freedom: three translations and one in rotation. For a multiple-wheel truck, each wheel had an angular degree of freedom which could include soil drag, rolling friction, and slip forces. The basic tire model was either a nonlinear spring with hysteretic damping or a multispring model.⁴² When the wheel was yawed, cornering forces were calculated. The landing gears were oleopneumatic with friction, spin-up/spring-back, and stiction. For the soil, Crenshaw's model⁴⁷ was modified. Several runway surface models were available that would allow for random elevations, various spacing of bomb damage repairs, and different forcing profiles on each wheel. Aerodynamic forces and moments due to fixed geometry and control surfaces, plus general thrust capabilities, were available. Integration techniques included either a fourth-order Adams-Moulton predictor-corrector, with a variable time step or a Taylor series. Simulations involved landing, takeoff, taxiing, and turning.

Observations

In a comparison of the simulation techniques reviewed, it would be easy to identify some models as being more accurate for a wider variety of conditions than other models. On the other hand, some models were developed to predict special phenomena, or simply to look at relative tradeoffs. In these cases a detailed model cannot be justified. However, there are several comments that can be made that apply to the general area of aircraft-surface simulation.

First, few of the simulations reviewed used a frequency-domain approach in which the outputs were rms values of vertical force or displacement. Most simulations were directed at predicting the dynamic response due to discrete events, or, in the case of operations on soil, the actual peak forces generated. This emphasis leads one to believe that the general thinking was that failure to successfully launch or retrieve an aircraft depends on avoiding a catastrophic occurrence, e.g., a landing gear failure due to a single BDR. Although catastrophic events must be avoided, high cycle fatigue may also cause a landing gear failure or structural failure of the airframe. High cycle fatigue may be due to as few as a couple dozen takeoffs and landings on an unpaved runway.

Second, several simulations considered the effect of asymmetric perturbations on the main gears. This required the introduction of extra degrees of freedom for the second main gear as well as for the airframe. In most cases an airframe roll degree of freedom was added, but not a lateral degree of freedom. This constrains the aircraft to roll about its longitudinal axis rather than translating laterally and rolling simultaneously. Although it might be argued that the effect is negligible, its significance would surely depend on the specific aircraft and the magnitude and phasing of the runway perturbations. None of the preceding reports attempted to justify the absence of an airframe lateral degree of freedom when the roll degree of freedom was included.

Third, many of the simulations used a point contact at the tire/surface interface, and a few of the simulations assumed that the tire stiffness was linear. Although tire properties are not as easily specified as metallic properties, they are available, and the radial stiffness is nonlinear. Any time-domain simulation attempting to predict the dynamic response due to surface irregularities in which the tire undergoes more than a small deflection should model the tire stiffness as a nonlinear spring. The relative merits of using a point follower are less obvious. If the slope of the surface is gradual and its

wavelength is long compared to the tire's footprint, a point follower is probably adequate. However, for steep, short obstacles and soil, the enveloping effect of the tire may be significant in determining the maximum forces developed.

Another phenomenon associated with the tire/surface interface is the tire's loss of runway contact due to a sharp dropoff in runway elevation. Although several of the models could handle the event because they simulated actual takeoff and landing, it was not clear whether or not some of the models, which might simulate landing or takeoff runout or high-speed taxiing, would allow the tire spring to expand beyond the undeformed tire radius (tension).

Also, it should be noted that several simulations attempted to model the soil by some combination of linear and/or nonlinear stiffness and damping elements. The need for such modeling is necessary to determine increased strut loads due to rutting and increased thrust needed to obtain takeoff speed. It is interesting to note that while several models include soil stiffness and damping elements, a soil mass element has not been considered. Since the apparent mass of the soil or even the runway structure may be equivalent to the mass of the wheel, the surface dynamics may contribute to the aircraft dynamics.

The last observation is in regard to the numerical integration techniques used. While fourth-order Runge-Kutta was the most common, several other techniques were used, e.g., predictor-correctors, finite differences, Taylor series, etc. The EASY, TAXIG, and NORTAX simulations gave the user a choice of several techniques. Some codes used a simple Taylor series technique because the number of calculations was large, but the frequencies were low. Runge-Kutta techniques were reserved for high-frequency oscillations. None of the documentation actually attempted to establish criteria for which technique should be used.

Table 1 summarizes a few of the more detailed aircraftsurface calculations.

Recommendations

Although there have been many worthwhile developments of dynamic simulations for aircraft-surface operations, there are several areas where future efforts could improve or add to present techniques. The following suggestions are given:

- 1) Develop frequency-domain simulations to determine high cycle effects.
- 2) Improve the tire model to model the traversing of short wavelength obstacles or dips more accurately.
- 3) Determine the accuracy of using a point-contact follower for the tire/ground interface.
- 4) Determine which numerical integration technique gives the most accurate and efficient results.
- 5) Determine the effect of neglecting the lateral degree of freedom when an airframe roll degree of freedom is included to simulate motion due to asymmetric obstacles.
- 6) Investigate the effect of wheel/runway loss of contact when negotiating obstacles.
- 7) Investigate the effect of surface apparent mass on aircraft-surface dynamics.
- 8) Develop small- and full-scale testing techniques to verify existing and future models. Some of the full-scale tests could be accomplished using the tire only or the tire and strut, e.g., verify tire models or the effect of loss of contact when negotiating obstacles.

Conclusions

A review of dynamic simulations of aircraft-surface operations revealed a wide variety of computer programs that predicted gear loads, structural response, and soil behavior when the aircraft traversed bomb damage repaired runways or maneuvered on soil. The simulation codes ranged from linear, single-degree-of-freedom models to nonlinear three-dimensional models with flexible airframe modes. Except for a few programs, the simulations involved numerical integration and sometimes an iteration. Many of the codes had been partially validated by test data or comparison to other codes.

Table 1 Simulation features and capabilities

	Degrees of freedom			· 			
	Airframe	Gears	Total No. of gears	Struts	Tires	Surfaces	Integration
NORTAX ⁵⁴	6 rigid 15 flexible	3 translation 1 rotational	5	Oleopneumatic Friction/ stiction	Nonlinear P-C ^a hysteresis	Irregular rigid Nonlinear yielding	Taylor Adams
LNDTAX2 ⁵²	Bounce Longitudinal Pitch Roll 15 flexible	1 Wheel spin Bounce Springback	3	Oleopneumatic Friction	Quadratic radial Nonlinear P-C	Regular rigid	Runge-Kutta
LATAX ⁵¹	Bounce Pitch Roll 8 Flexible	Bounce	3	Oleopneumatic Friction	Nonlinear P-C linear damping	Regular rigid	Runge-Kutta
FILTER1 ⁴⁹	6 rigid	Bounce	3	Oleopneumatic Friction	Quadratic radial	Irregular rigid Nonlinear yielding	Taylor
TURN ⁴⁷	6 rigid	Bounce Flexible	3	Oleopneumatic	P-C Quadratic radial	Nonlinear yielding	Runge-Kutta Adams
TAX2 ²⁸	Bounce Longitudinal Pitch Roll 30 flexible	Bounce	3	Oleopneumatic	Linear P-C	Irregular rigid	Taylor

^aPoint contact.

Based on the programs reviewed, there seemed to be less effort to consider frequency-domain analysis in an attempt to predict high cycle fatigue than to use time-domain analysis to predict catastrophic failure, especially in the past 10 years. Although some simulations included a detailed tire model, most used a point contact to model the tire/ground interface. Last, several numerical integration techniques were used. Numerical instability and long running times were cited as problems by more than one author.

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